INFLUENCE OF HYDRODYNAMICS ON THE THERMODIFFUSION PROCESS OF SEPARATION OF GAS MIXTURES

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Thermodiffusion, as a typical case of an irreversible process of redistribution of molecules of a mixture in the direction of the temperature gradient, is extensively employed in the technological process of separation of gas mixtures and isotopes. For a binary mixture the flux of the enriched component in such separation is expressed by the relation

$$\mathbf{I} = \alpha\rho\mathbf{U} - \rho D_{12}[\mathrm{grad}\ \alpha - \beta_t\alpha(1-\alpha)\ \mathrm{grad}\ \ln T],$$

where U is the velocity; ρ is the density; T is the temperature; α is the mass concentration; D_{12} is the binary diffusion coefficient; β_t is the thermodiffusion ratio. For an elementary process of thermodiffusional separation the velocity U equals zero, and in this case the separation coefficient r_w is determined from the above relation with **I** = 0:

$$r_w = \frac{\alpha_w}{1-\alpha_w} \Big/ \frac{\alpha_\infty}{1-\alpha_\infty} = \left(\frac{T_w}{T_\infty}\right)^{\beta_t}.$$

For U ≠ 0, hydrodynamic processes become decisive in thermodiffusional separation. Motion of the mixture can essentially alter the efficiency of the separation process. This can be clearly traced on the example of the classic Clusius–Dickel thermogravitation column. The latter consists of two vertically placed surfaces, the space between which is filled with the mixture to be separated. The surfaces are maintained at different temperatures, as a result of which the elementary process of thermodiffusional separation occurs in any cross section of the column in the horizontal direction owing to the temperature difference. At the same time, convective motion develops in the column owing to the difference in the densities of the mixture near the cold and hot surfaces: The less-heated gas descends while the more-heated gas rises. Opposite motions take place in the column, the result of which is the manifold multiplication of the elementary process of separation. At the same time, the nonuniform temperature distribution over the surface of the column, the instability of the opposite flows, and other hydrodynamic phenomena can considerably reduce the efficiency of the idealized separation process discussed above.

The influence of all these hydrodynamic phenomena on the process of thermodiffusional separation of gas mixtures can be investigated on the basis of exact solutions of the complete system of Navier–Stokes equations. The number of such solutions is extremely limited, however. The results of such investigations for plane and axisymmetric thermogravitation columns can be found in [1]. The influence of thermodiffusion on the spherical expansion of a binary mixture of viscous heat-conducting gas was investigated in [2]. With arbitrary motion of the gas mixtures it becomes a very complicated problem to obtain such solutions. In this case it is advisable to use various asymptotic representations of the Navier–Stokes equations. The results of such investigations for thin shock and boundary layers are presented below. Such an approach, in defining the essence of the phenomenon, makes it possible, even in the first approximation, to exclude any extraordinary influence of longitudinal gradients of the stream parameters on the flow in thin layers in the dissipative terms of the Navier–Stokes equations.

1. Let us consider the supersonic flow over a blunt body of a stream of a binary mixture of nonreacting gases when processes of thermo- and barodiffusion are present. We assume that the disturbed region of flow is described by a two-layer model of a viscous shock layer [3]. In the plane (ν = 0) and axisymmetric (ν = 1) cases the corresponding system of equations is written in the dimensionless form

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$$\frac{\partial}{\partial x}(r^\nu \rho u) + \frac{\partial}{\partial y}(r^\nu \rho v) = 0, \tag{1.1}$$

$$\rho\left(u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y}\right) = -\frac{\varepsilon_1}{2}\frac{dp_w}{dx} + \chi\frac{\partial}{\partial y}\left(\mu\frac{\partial u}{\partial y}\right), \quad \frac{\partial p}{\partial y} = 2k\rho u^2,$$

$$\rho\left(u\frac{\partial \alpha_1}{\partial x} + v\frac{\partial \alpha_1}{\partial y}\right) = \frac{\partial j_1}{\partial y},$$

$$\rho\left(u\frac{\partial H}{\partial x} + v\frac{\partial H}{\partial y}\right) = \frac{\partial q}{\partial y} + 2\chi\frac{\partial}{\partial y}\left(\mu u\frac{\partial u}{\partial y}\right), \quad p = \rho RT, \quad \alpha_1 + \alpha_2 = 1,$$

where uU_∞ and $\varepsilon_1 vU_\infty$ are the velocity components along the coordinates xL and $\varepsilon_1 yL$ connected with the surface of the body; r is the dimensionless distance from the axis of symmetry to the surface of the body; L is the characteristic length; k/L is the curvature of the surface; $\rho\rho_\infty/\varepsilon_1$ is the density; $p\rho_\infty U^2{}_\infty/2$ is the pressure; $TU^2{}_\infty/2c_p$ is the temperature; $hU^2{}_\infty/2$ is the enthalpy; $H = h + u^2$; $\varepsilon_1 = (\gamma - 1)/2\gamma$; $\chi = (\varepsilon_1 \mathrm{Re}_0)^{-1}$; $\mathrm{Re}_0 = \rho_\infty U_\infty L/\mu_0$; μ_0 is the coefficient of viscosity at the stagnation temperature T_0; $R = 2\varepsilon_1(c_{p1}\alpha_1 + c_{p2}\alpha_2\varepsilon_2/\varepsilon_1)$ is the gas constant; c_p is the specific heat at constant pressure; γ is the ratio of specific heats; $j_1\rho_\infty \cdot U_\infty$ and $q\rho_\infty U^3{}_\infty/2$ are the diffusional and heat fluxes, which are defined by the equalities

$$j_1 = \chi\frac{\mu}{\mathrm{Sc}}\left\{\frac{\partial \alpha_1}{\partial y} + \alpha_1(1-\alpha_1)\left[\frac{m_2 - m_1}{m}\frac{\partial \ln p}{\partial y} + \beta_t\frac{\partial \ln T}{\partial y}\right]\right\}, \tag{1.2}$$

$$q = \chi\frac{\mu c_p}{\mathrm{Pr}}\frac{\partial T}{\partial y} + j_1(h_1 + h_2);$$

$m = m_1\alpha_1 + m_2\alpha_2$ is the molecular weight of the mixture; Sc and Pr are the Schmidt and Prandtl numbers. Quantities pertaining to the component with the higher molecular weight are denoted by the index 1 and for the lower — by the index 2, the index ∞ determines parameters in the undisturbed stream, and w determines those at the surface of the body.

The dependences of the thermodynamic and transfer coefficients on the parameters of state of the mixture were taken in accordance with experimental data [4]. An argon–helium mixture was used in all the calculations below.

If the temperature of the surface of the body is considerably less than the stagnation temperature, attachment boundary conditions are satisfied at the surface of the body:

$$u = v = 0, \; T = T_w, \; j_{1w} = 0. \tag{1.3}$$

The last condition means that a diffusional flux through the surface of the body is absent, and it will be valid only in the case when the injection or suction of material through the surface of the body is absent.

At the outer boundary of the shock layer the system (1.1) satisfies the generalized Rankine–Hugoniot conditions which, in the case of the flow of a binary mixture, can be written in the form (see [3], for example)

$$\rho v = -\sin\sigma, \; p = \sin^2\sigma, \; \sin\sigma(\cos\sigma - u) = \chi\mu\partial u/\partial y, \tag{1.4}$$

$$\sin\sigma(\alpha_{1\infty} - \alpha_1) = j_1, \; \sin\sigma(H_\infty - H) = q + 2\chi\mu u\partial u/\partial y,$$

where σ is the angle of inclination of the shock, which coincides with the inclination of the generatrix of the surface of the body in a first approximation.

The flow structure in the viscous shock layer (below we shall call this the inner region) is fully described by the system of equations (1.1) with the boundary conditions (1.3) and (1.4) at the surface of the body and at the outer boundary of this layer. In the numerical solution of this system we used independent variables of the Mises type and applied a simple finite-difference scheme, of the type of Keller's "box" scheme [5], which is more economical than the latter and has the same (second) order of approximation in both variables.

The flow in the region of the compression shock (we shall call it the outer region) in the two-layer scheme of a viscous shock layer is described by a system of ordinary differential equations [2]. In our case, it has the form

$$\partial\hat{y}/\partial\eta = \hat{\mu}/(\mathrm{Re}_0\sin\sigma), \; \partial\hat{v}/\partial\eta = (3/4)(1 - \hat{v} - \hat{p}/2), \tag{1.5}$$

$$\partial\hat{\alpha}_1/\partial\eta = [\mathrm{Sc}(\hat{\alpha}_{1\infty} - \hat{\alpha}_1) - A]/B, \; \partial\hat{T}/\partial\eta = (\mathrm{Pr}/c_p)(\hat{H}_\infty - \hat{H} - C),$$

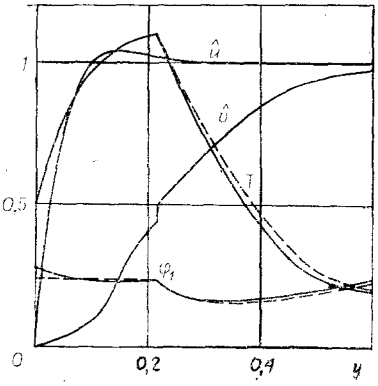

Fig. 1

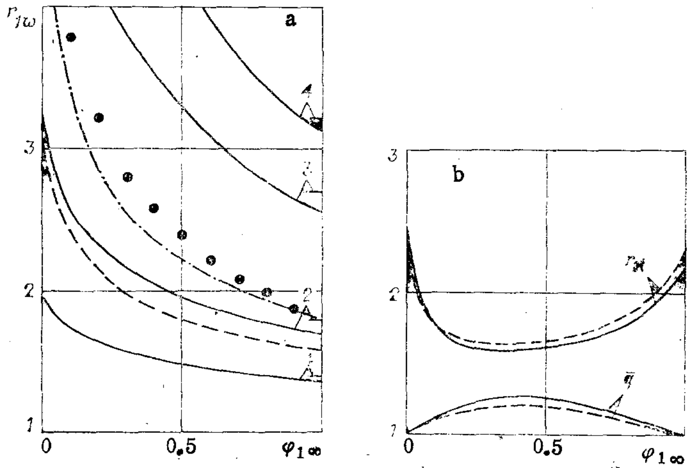

Fig. 2

where

$$A=\hat{\varphi}_1\left(\frac{\partial\ln\hat{v}}{\partial\eta}-\frac{\partial\ln\hat{T}}{\partial\eta}\right)+\hat{\alpha}_1(1-\hat{\alpha}_1)\beta_t\frac{\partial\ln\hat{T}}{\partial\eta},\quad B=1-\frac{2\hat{\varphi}_1}{\hat{R}}\times$$

$$\times(\hat{c}_{p1}\varepsilon_1-\hat{c}_{p2}\varepsilon_2),\quad C=\hat{u}\frac{\partial\hat{u}}{\partial\eta}\cos^2\sigma+\frac{4}{3}\hat{v}\frac{\partial\hat{v}}{\partial\eta}\sin^2\sigma+(\hat{\alpha}_{1\infty}-\hat{\alpha}_1)\times$$

$$\times\left(\hat{c}_{p1}-\hat{c}_{p2}+\beta_t\frac{m_2}{m_1m_2}\hat{R}\right)\hat{T},\quad \hat{\varphi}_1=\frac{\hat{\alpha}_1}{\omega}(1-\hat{\alpha}_1)(1-\omega)[1-(1-\omega)\hat{\alpha}_1],$$

$$\hat{u}=1-(1-\hat{u}_s)\exp(-\eta).$$

Here $\hat{u}U_\infty\cos\sigma$ and $-\hat{v}U_\infty\sin\sigma$ are the velocity components along the coordinates $\hat{x}L$ and $\hat{y}L$, respectively; $\hat{u}_s$ is the value of $\hat{u}$ at the outer boundary of the inner region of flow; $\hat{p}\rho_\infty U^2_\infty$ is the pressure; $\hat{\rho}\rho_\infty$ is the density; $\hat{T}U^2_\infty/2c_{p\infty}$ is the temperature; $\hat{\mu}\mu_0$ is the viscosity coefficient; $\hat{H}=\hat{h}+\hat{u}^2\cos^2\sigma+\hat{v}^2\sin^2\sigma$; $\hat{H}_\infty=1+2[(\gamma_\infty-1)M^2_\infty]^{-1}$; M_∞ is the Mach number; $\hat{c}_p=\hat{c}_{p1}\hat{\alpha}_1+\hat{c}_{p2}(1-\hat{\alpha}_1)$; $\hat{R}=2\varepsilon_1[\hat{c}_{p1}\hat{\alpha}_1+\hat{c}_{p2}(1-\hat{\alpha}_1)\varepsilon_2/\varepsilon_1]$; $\gamma_\infty=(1-2\varepsilon_\infty)^{-1}$; $\omega=m_2/m_1$; $\hat{p}=\hat{\rho}\hat{R}\hat{T}\sin^{-2}\sigma$; $\hat{\rho}\hat{v}=1$; parameters pertaining to the outer region of flow are marked by the symbol ^.

An analysis of the system of equations (1.5) shows that smooth joining of the outer solution with the inner one is possible only when the solution in the inner region is found in the second approximation. In the present work the solution in the viscous shock layer was found in the first approximation. Therefore, by analogy with what was done in [3], the conditions for joining the quantities $\hat{u}$, $\partial\hat{v}/\partial\hat{y}$, $\hat{H}$, and $\hat{\alpha}_1$ were used to find the solution in the outer region. Numerical integration of the system (1.5) was carried out by the Runge-Kutta method.

The results of numerical calculations with $Re_0=50$, $M_\infty=5$, $t_w=T_w/T_0=0.5$, $\omega=0.1$, and $\varphi_{1\infty}=0.24$ for a sphere of radius L are presented as an example in Fig. 1, where the variations of the quantity $\hat{u}=u/\cos\sigma$, the normal velocity component $\hat{v}$, the temperature T, and the local concentration $\varphi_1 = \omega\alpha_1[1 - (1 - \omega)\alpha_1]^{-1}$ of the heavy component on the zero streamline are given (solid lines for $\beta_t \neq 0$, dashed lines for $\beta_t = 0$).

As the calculations show, in a shock wave front the role of thermodiffusion in the process of separation of a mixture proves to be relatively small. Here the redistribution of components of the mixture will be due mainly to barodiffusional separation. In the adopted two-layer model of a thin viscous shock layer of the first approximation the influence of barodiffusion in the region of the transition through the shock on the redistribution of components of the mixture is analogous to that for the one-dimensional case [2, 6]. Allowance for the three-dimensionality of the flow on the process of separation of a mixture at low Reynolds numbers Re_o was investigated in [7, 8].

Thermodiffusional separation of the mixture becomes decisive near the surface of the body at low values of t_w, when the temperature gradients in the wall region of flow become large. Barodiffusional separation of a mixture in the wall region of the viscous shock layer is absent in the adopted approximation. The latter fact is connected with the conditions adopted here for joining the regions in the two-layer model of a viscous shock layer. With smooth joining this effect results in an additional increase in the concentration of heavy particles at the wall of the body at moderate values of Re_o. This is indicated both by experimental [7] and by calculated data [8] obtained using the complete system of Navier—Stokes equations.

In hypersonic flow over bodies by a viscous gas the number of hydrodynamic similarity criteria determining the flow is rather large. They include Re_o, M_∞, Sc, Pr, the temperature factor t_w, the initial concentration $\alpha_{1\infty}$, etc. In the regime of hypersonic stabilization at $M_\infty >> 1$ the Mach number is eliminated from the system of similarity criteria, in connection with which all the calculations below were made for the same Mach number ($M_\infty = 5$). As for the other similarity criteria, in the problems of separation of gas mixtures being considered here the important ones among them will be Re_o, t_w, and $\alpha_{1\infty}$ for thermodiffusional separation near the surface of a body and Re_o and $\alpha_{1\infty}$ for barodiffusional separation in the disturbed region of flow, as the calculations show.

The results of numerical calculations of the separation coefficients for these two processes are presented in Fig. 2. In Fig. 2a we give the maximum values of the separation coefficient r_{1w} at the surface of the body as a function of the initial molar concentration $\varphi_{1\infty}$ of the heavy component for $Re_o = 50$ (solid lines) and values of $t_w = 0.25$, 0.1, 0.02, and 0.01 (curves 1-4, respectively). The separation coefficient r_{1w} grows as the temperature factor t_w and the initial concentration $\varphi_{1\infty}$ decrease. As $\varphi_{1\infty}$ approaches one, when the thermodiffusion coefficient β_t can be taken as constant, its values is close to the value $r_{1w} = t_w^{-\beta_t(\varphi_{1\infty})}$ determined by the elementary process of thermodiffusional separation (straight-line segments in Fig. 2a). The degree of separation of the mixture near the surface of the body hardly varies with an increase in the Reynolds number $Re_o > 50$ and decreases as this criterion decreases (the values of r_{1w} for $Re_o = 10$ and $t_w = 0.1$ are given by a dashed line in Fig. 2a). A similar result was obtained in [9] in an analysis of the thermodiffusion process of separation on the basis of kinetic equations.

The maximum values of the separation coefficient r_{2s} in the shock wave front as a function of the initial concentration for $Re_o = 50$ are given in Fig. 2b (solid line). As already mentioned, the enrichment of the mixture with the light component in this region is due to the barodiffusion process and, therefore, as the calculations show, it hardly depends on the value of the temperature factor t_w. The efficiency of this process proves to be higher at low relative concentrations of the components of the mixture and, with the exception of small $\varphi_{1\infty}$ it increases as the Reynolds number Re_o decreases (the values of r_{2s} for $Re_o = 10$ are given by a dashed line in Fig. 2b).

It should be noted that at moderate Reynolds numbers the diffusion processes result in an increase in the heat flux q to the body with hypersonic flow over it. Its value at the critical point of the body as a function of the initial concentration $\varphi_{1\infty}$ is given in Fig. 2b for $t_w = 0.1$ and $Re_o = 50$ and 10 (solid and dashed lines, respectively). Here this phenomenon is due to thermodiffusion. The barodiffusion process results in a still larger increase in the heat flux to the body. Measurements made in [7] showed that for $Re_o = 80$ and $t_w = 1$ the heat flux at the critical point of a body over which a nitrogen—hydrogen mixture flows exceeds fivefold the corresponding flux when pure nitrogen or hydrogen flows over the same body.

2. The investigation of flows of binary gas mixtures in boundary layers when thermodiffusion is present was confined to self-similar solutions. In the Dorodnitsyn–Lees variables,

$$\xi = \int_0^x r_w^{2\nu} \rho_w \mu_w U_\infty dx, \quad \eta = \frac{r_w^\nu U_\infty}{(2\xi)^{1/2}} \int_0^y \rho dy$$

the initial system of equations in this case is written in the form

$$(Nf'')' + iff'' + \beta\left[\frac{\varepsilon}{\varepsilon_\infty}(g - \beta_\infty f'^2) - (1 - \beta_\infty) f'^2\right] = 0, \qquad (2.1)$$
$$j_1' + i\alpha_1' f = 0, \quad q' + ifg' + 2\beta_\infty (Nf'f'')' = 0.$$

The dimensionless diffusion and heat fluxes j_1 and q appearing in it are defined as follows:

$$j_1 = -\frac{N}{\mathrm{Sc}}\left[\alpha_1' + \alpha_1(1 - \alpha_1)\beta_t(g - \beta_\infty f'^2)\right](g - \beta_\infty f'^2)^{-1},$$
$$q = \frac{N}{\mathrm{Pr}}(g' - 2\beta_\infty f'f'') + \frac{j_1}{c_p}\left(c_{p1} - c_{p2} + \beta_t \frac{m_2}{m_1 m_2} R\right)(g - \beta_\infty f'^2).$$

Here $f' = u/U_\infty$, $g = H/H_\infty$, $N = \rho\mu/\rho_W\mu_W$, $\beta_\infty = U^2_\infty/2H_\infty$, i and β are the Falker–Skan similarity parameters; a prime denotes a derivative with respect to η; parameters at the outer limit of the boundary layer are marked by the index ∞.

The boundary conditions of the problem are written in the form

$$\eta = 0{:}\ f = f_w = \text{const},\ f' = 0,\ g = t_w,\ (1 - 2\alpha_1)j_1 = f_w(1 - \alpha_1)\alpha_1, \qquad (2.2)$$
$$\eta = \eta_\infty{:}\ f' = 1,\ g = 1,\ \alpha_1 = \alpha_{1\infty}.$$

The two main cases of i = 1 and 0 are distinguished in an analysis of self-similar solutions. For $\beta_\infty = 0$ the first case determines the flow in the boundary layer in the vicinity of the critical point of a wedge with an angle of taper $\beta = 2n/(n + 1)$ ($U_\infty \sim x^n$). In the hypersonic approximation ($\beta_\infty \to 1$) this case corresponds to flow in the boundary layer on a body with a power-law dependence of the pressure p on the x coordinate ($p \sim x^n$). In this case the similarity parameter is $\beta = -\frac{\gamma - 1}{\gamma}\frac{n}{n + 1}$.

The case of i = 0 with $\beta_\infty = 0$ and all values of β corresponds to boundary-layer flow for potential flows with $U_\infty \sim x^{-1}$. Depending on the sign of U_∞, it will be either flow from a source or flow toward a sink and can be treated as flow in widening or narrowing channels with plane walls. In this case, in the absence of suction or injection ($f_W = 0$), the system of equations (2.1) with the boundary conditions (2.2) is simplified and admits of the particular integrals

$$(Nf'')' + \beta[\alpha(g - \beta_\infty f'^2) - (1 - \beta_\infty)f'^2] = 0, \qquad (2.3)$$
$$j_1 = 0, \quad \frac{N}{\mathrm{Pr}} g' + 2\beta_\infty N\left(1 - \frac{1}{\mathrm{Pr}}\right)f'f'' = \text{const}.$$

The particular case of 2i − β = 0 and $\beta_\infty = 0$ corresponds to boundary-layer flow for potential flows $U_\infty \sim e^{ax}$, where a is a positive or negative constant. By analogy with the preceding case, such flows can occur in widening or narrowing channels with exponential generatrices.

The system of equations (2.1) with different values of the parameters i, β, and β_∞ and with the boundary conditions (2.2) was integrated numerically using a finite-difference scheme of second-order accuracy. Here the solution of the finite-difference system of equations was found by the trial-run method [10] with the subsequent application of an iteration process. Some results of the numerical calculations are given below.

The distribution of the mass concentration α_1 of the heavy component in the boundary layer for $\alpha_{1\infty} = 0.76$ ($\varphi_{1\infty} = 0.24$) is presented in Fig. 3. The dash–dot lines ($\beta_\infty = 0$) pertain to the case of i = 0 and 2i − β = 0 while the solid ($\beta_\infty = 0$) and dashed ($\beta = 0$, $\beta_\infty = 0.89$) lines pertain to the case of i = 1. For $\beta_\infty = 0$, the temperature factor in these calculations was kept constant, $t_W = 0.1$, and for $\beta_\infty = 0.89$ it was varied.

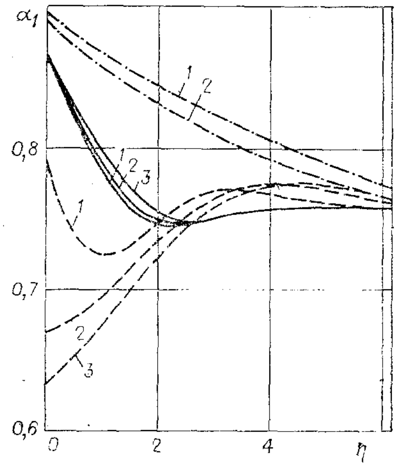

Fig. 3

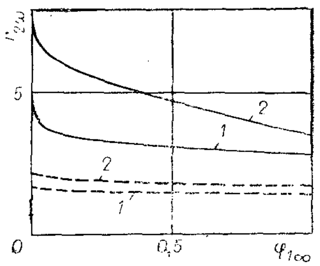

Fig. 4

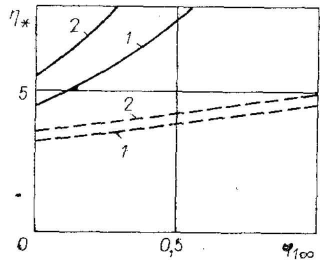

Fig. 5

For $i = 1$ and $\beta_\infty = 0$ (solid lines in Fig. 3), the thermodiffusional separation of a mixture near the surface of a cooled body increases in the transition from accelerated ($\beta > 0$) to decelerated ($\beta < 0$) potential flows. This is indicated by the relations $\alpha_1(\eta)$ presented in Fig. 3 and corresponding to plane flow near the critical point ($\beta = 1$, curve 1), longitudinal flow over a plate ($\beta = 0$, curve 2), and decelerated plane flow with a preseparation velocity profile ($\beta = -0.5$, curve 3). We note here that axisymmetric flow near the critical point occurs for $\beta = 0.5$. For attached flows, deceleration of the motion results in an increase in the mass concentration of the heavy component in the boundary layer by several percent. The mass concentration α_{1w} at a cooled surface in these regimes hardly varies with variation of β, in connection with which the maximum values of the separation coefficient r_{1w} in all the cases considered above will coincide with the values of r_{1w} obtained earlier for the axisymmetric critical point in hypersonic flow over a blunt body (see Fig. 2a). For $\beta < -0.5$, return flows appear in the boundary layer. Further stream deceleration in these regimes will be limited by the conditions of stability of such flows.

Flows in channels with straight ($i = 0$) or exponential ($2i - \beta = 0$) generatrices are of particular interest in separation problems. The degree of separation proves to be considerably higher in them. This is indicated by the $\alpha_1(\eta)$ distribution corresponding to the case of $i = 0$ presented in Fig. 3 (dash–dot line 1). The increase in the mass concentration of the heavy component in such flow is connected with the increasing temperature gradient owing to the decrease in the characteristic transverse size of the flow. In this case the solution of the system of equations (2.1) is equivalent to the corresponding inner solution of the problem with $i = 1$ and $\beta \to \pm\infty$. The same degrees of separation as above occur in channels with an exponential generatrix ($2i - \beta = 0$). According to the numerical calculations, the concentration profiles across the boundary layers in these flows practically coincide. Only when developed zones of return flows are present does the degree of separation in such channels start to decrease (dash–dot line 2 in Fig. 3).

The maximum values of the separation coefficient r_{1w} at the surfaces of the channels under consideration as a function of the initial concentration $\varphi_{1\infty}$ of the heavy component for $t_w = 0.1$ are presented in Fig. 2a (dash–dot line). For all values of $\varphi_{1\infty}$ the efficiency of separation of gas mixtures in channels proved to be higher and is determined by the elementary process of thermodiffusional separation (circles in Fig. 2a). The latter follows from the second equation of (2.3) with $\beta_t = \beta_{t\infty} = \mathrm{const}$:

$$r_{1w} = [(1 - \beta_\infty)/t_w]^{\beta_{t\infty}}.$$

At low concentrations of one of the components of the mixture, when the thermodiffusion coefficient β_t varies little in the boundary layer, the numerical values of r_{1w} obtained coincide with the analytical values.

The case of hypersonic flow of a gas mixture ($\beta_\infty \to 1$) offers the greatest practical interest; in this case the concentration distribution across the boundary layer for $t_w < 1$ has a nonmonotonic character. This is connected with the fact that as the Mach number $M_\infty = [2\beta_\infty/(\gamma - 1)(1 - \beta_\infty)]^{1/2}$ increases, the ratio of the gas temperature at the outer limit of the boundary layer to the stream stagnation temperature decreases:

$$T_\infty = T_0\left(1 + \frac{\gamma - 1}{2} M_\infty^2\right)^{-1} = T_0(1 - \beta_\infty).$$

The concentration of the heavy component will increase in this region, just as in the wall region. The relation $\alpha_1(\eta)$ for $t_w = 0.1$ is given in Fig. 3 (dashed line 1).

In the hypersonic case the degree of separation of gas mixtures can increase considerably for $t_w > 1$. A considerable decrease in the gas temperature at the outer limit of the boundary layer due to hypersonic stream acceleration, in a nozzle with a simultaneous increase in wall temperature, for example, leads to a considerable increase in the temperature gradients in the boundary layer and thereby intensifies the thermodiffusional separation of the mixture. As the calculations show, such separation becomes efficient even for $\beta_\infty = 0.89$ (the dashed lines 2 and 3 in Fig. 3 correspond to $t_w = 1$ and 2).

The maximum values of the separation coefficient r_{2w} in the hypersonic case with $\beta_\infty = 0.89$ and 0.99 (dashed and solid lines, respectively) are presented in Fig. 4. Curves 1 pertain to the case of $t_w = 1$ and curves 2 to the case of $t_w = 2$. As M_∞ increases, the degree of separation of gas mixtures in the hypersonic case becomes very high. For the elementary process of thermodiffusional separation such values of r_{2w} can be obtained only for very low values of the temperature factor t_w.

For the determination of the flow regions in a hypersonic boundary layer enriched with the light and heavy components of the mixture, in Fig. 5 we present the coordinate η_* at which $\varphi_1 = \varphi_{1\infty}$ (notation the same as in Fig. 4). These results show that as the Mach number increases, an ever greater part of the boundary layer, the effective thickness of which increases in the process, as is known, is enriched with the light component. In connection with the latter data, it must also be mentioned that, in contrast to the classic thermodiffusion column, in which counterflow motion can be unstable, the flow in a hypersonic boundary layer with negative pressure gradients (flow in a nozzle, for example) will always be stable.

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LIQUID TRAPPING ON CYLINDER EXTRACTION

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It is important to know the thickness of the film of liquid formed on a cylindrical body, for example in depositing insulation on wires and also in the production of glass and synthetic fibers. The theory of [1-3] is restricted to low extraction velocities. The approach considered below is applicable to a very wide velocity range.

1. Consider a cylinder of radius R extracted at a constant velocity U from a sufficiently large volume of liquid (Fig. 1). The thickness of the film remaining on the surface is determined by the interaction between the internal friction, the mass forces, and the surface tension. The effect of these forces on the trapping are determined primarily by the extraction speed and the properties of the medium.

The liquid in the film is simultaneously extracted by the cylinder and flows under gravity back into the bath. Therefore, at the surface of the film there should be a stagnation line, where the flow direction reverses. The stream lines passing through this separate the part of the liquid carried by the cylinder from the rest in the bath. We write the equations of motion for each of these regions and find the condition for linking up the solutions.

2. We set the z axis along the flow parallel to the cylinder axis, while the r axis is perpendicular to it and passes through the stagnation line. The region of entrainment is bounded from below by a plane perpendicular to the axis of the cylinder and passing through the stagnation line, while upwards it passes into the region of constant film thickness $h_0 = \xi_0 - R$. Physical considerations show that the characteristic dimension L of this region considerably exceeds h_0, i.e., $h_0/L = \varepsilon \ll 1$.

We write the Navier-Stokes equations and the boundary conditions for the extraction region:

$$u\frac{\partial u}{\partial z}+v\frac{\partial u}{\partial r}=-\frac{1}{\rho}\frac{\partial p}{\partial z}-g+\nu\left(\frac{\partial^2 u}{\partial r^2}+\frac{1}{r}\frac{\partial u}{\partial r}+\frac{\partial^2 u}{\partial z^2}\right),\tag{2.1}$$

$$u\frac{\partial v}{\partial z}+v\frac{\partial v}{\partial r}=-\frac{1}{\rho}\frac{\partial p}{\partial r}+\nu\left(\frac{\partial^2 v}{\partial r^2}-\frac{v}{r^2}+\frac{1}{r}\frac{\partial v}{\partial r}+\frac{\partial^2 v}{\partial z^2}\right),$$

$$\frac{\partial u}{\partial z}+\frac{\partial v}{\partial r}+\frac{v}{r}=0;$$

$$\frac{\partial u}{\partial r}+\frac{\partial v}{\partial z}+\frac{2d\xi}{dz}\left[1-\left(\frac{d\xi}{dz}\right)^2\right]^{-1}\left(2\frac{\partial v}{\partial r}+\frac{v}{r}\right)=0 \quad \text{at} \quad r=\xi,\tag{2.2}$$

$$p-p_0+\sigma\left[1+\left(\frac{d\xi}{dz}\right)^2\right]^{-\frac{1}{2}}\left\{\frac{d^2\xi}{dz^2}\left[1+\left(\frac{d\xi}{dz}\right)^2\right]^{-1}-\frac{1}{\xi}\right\}=$$
$$=2\mu\left[1-\left(\frac{d\xi}{dz}\right)^2\right]^{-1}\left\{\left[1+\left(\frac{d\xi}{dz}\right)^2\right]\frac{\partial v}{\partial r}+\frac{v}{r}\right\} \quad \text{at} \quad r=\xi;$$

$$u=U,\ v=0 \quad \text{at} \quad r=R;\tag{2.3}$$

$$v=u\frac{d\xi}{dz} \quad \text{at} \quad r=\xi,\tag{2.4}$$

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